North Carolina Department of Public Instruction
INSTRUCTIONAL SUPPORT TOOLS
FOR ACHIEVING NEW STANDARDS

7th Grade Mathematics ● Unpacked Contents
For the new Standard Course of Study that will be effective in all North Carolina schools in the 2018-19 School Year.

This document is designed to help North Carolina educators teach the 7th Grade Mathematics Standard Course of Study. NCDPI staff are continually updating and improving these tools to better serve teachers and districts.

What is the purpose of this document?
The purpose of this document is to increase student achievement by ensuring educators understand the expectations of the new standards. This document may also be used to facilitate discussion among teachers and curriculum staff and to encourage coherence in the sequence, pacing, and units of study for grade-level curricula. This document, along with on-going professional development, is one of many resources used to understand and teach the NC SCOS.

What is in the document?
This document includes a detailed clarification of each standard in the grade level along with a *sample* of questions or directions that may be used during the instructional sequence to determine whether students are meeting the learning objective outlined by the standard. These items are included to support classroom instruction and are not intended to reflect summative assessment items. The examples included may not fully address the scope of the standard. The document also includes a table of contents of the standards organized by domain with hyperlinks to assist in navigating the electronic version of this instructional support tool.

How do I send Feedback?
Link for: Feedback for NC's Math Resource for Instruction We will use your input to refine our unpacking of the standards. Thank You!

Just want the standards alone?
Link for: NC Mathematics Standards

North Carolina 7th Grade Standards

Standards for Mathematical Practice

Ratio and Proportional Relationships	The Number System	Expressions & Equations	Geometry	Statistics & Probability
Analyze proportional relationships and use them to solve real-world and mathematical problems. **NC.7.RP.1** **NC.7.RP.2** **NC.7.RP.3**	*Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.* **NC.7.NS.1** **NC.7.NS.2** **NC.7.NS.3**	*Use properties of operations to generate equivalent expressions.* **NC.7.EE.1** **NC.7.EE.2** *Solve real-world and mathematical problems using numerical and algebraic expressions, equations, and inequalities.* **NC.7.EE.3** **NC.7.EE.4**	*Draw, construct, and describe geometrical figures and describe the relationships between them.* **NC.7.G.1** **NC.7.G.2** *Solve real-world and mathematical problems involving angle measure, area, surface area, and volume.* **NC.7.G.4** **NC.7.G.5** **NC.7.G.6**	*Use random sampling to draw inferences about a population.* **NC.7.SP.1** **NC.7.SP.2** *Make informal inferences to compare two populations.* **NC.7.SP.3** **NC.7.SP.4** *Investigate chance processes and develop, use, and evaluate probability models.* **NC.7.SP.5** **NC.7.SP.6** **NC.7.SP.7** **NC.7.SP.8**

Standards for Mathematical Practice

Practice	Explanations and Examples
1. Make sense of problems and persevere in solving them.	In grade 7, students solve problems involving ratios and rates and discuss how they solved the problems. Students solve real world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?".
2. Reason abstractly and quantitatively.	In grade 7, students represent a wide variety of real world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.
3. Construct viable arguments and critique the reasoning of others.	In grade 7, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (i.e. box plots, dot plots, histograms, etc.). The students further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like "How did you get that?", "Why is that true?", "Does that always work?". They explain their thinking to others and respond to others' thinking.
4. Model with mathematics.	In grade 7, students model problem situations symbolically, graphically, tabularly, and contextually. Students form expressions, equations, or inequalities from real world contexts and connect symbolic and graphical representations. Students explore covariance and represent two quantities simultaneously. They use measures of center and variability and data displays (i.e. box plots and histograms) to draw inferences, make comparisons and formulate predictions. Students use experiments or simulations to generate data sets and create probability models. Students need many opportunities to connect and explain the connections between the different representations. They should be able to use all of these representations as appropriate to any problem's context.
5. Use appropriate tools strategically.	Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. For instance, students in grade 7 may decide to represent similar data sets using dot plots with the same scale to visually compare the center and variability of the data. Students might use physical objects or applets to generate probability data and use graphing calculators or spreadsheets to manage and represent data in different forms.
6. Attend to precision.	In grade 7, students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students define variables, specify units of measure, and label axes accurately. Students use appropriate terminology when referring to rates, ratios, probability models, geometric figures, data displays, and components of expressions, equations or inequalities.
7. Look for and make use of structure.	Students routinely seek patterns or structures to model and solve problems. Students apply properties to generate equivalent expressions (i.e. $6 + 2x = 3(2 + x)$ by distributive property) and solve equations (i.e. $2c + 3 = 15$, $2c = 12$ by subtraction property of equality), $c = 6$ by division property of equality). Students compose and decompose two- and three-dimensional figures to solve real world problems involving scale drawings, surface area, and volume. Students examine tree diagrams or systematic lists to determine the sample space for compound events and verify that they have listed all possibilities.
8. Look for and express regularity in repeated reasoning.	In grade 7, students use repeated reasoning to understand algorithms and make generalizations about patterns. During multiple opportunities to solve and model problems, they may notice that $a/b \div c/d = ad/bc$ and construct other examples and models that confirm their generalization. They extend their thinking to include complex fractions and rational numbers. Students formally begin to make connections between covariance, rates, and representations showing the relationships between quantities. They create, explain, evaluate, and modify probability models to describe simple and compound events.

Return to: Standards

Analyze proportional relationships and use them to solve real-world and mathematical problems.

NC.7.RP.1 Compute unit rates associated with ratios of fractions to solve real-world and mathematical problems.

Clarification

This standard asks students to understand the concepts of a unit rate in proportional relationships. This concept will allow students to write equations, graph and compare proportional relationships.

In 6th grade, students learned to find the multiplicative relationships within a ratio, the rate, and they explored the concepts of independent and dependent variables. Students also learned that equivalent ratios also had equivalent rates.

In 7th grade, students build on this understanding to:

- Find the appropriate rate based on context.
- Rewrite any rate as a unit rate.
- Know that a rate can be used to express all of its associated equivalent ratios.

Ratios in 7th grade can include fractions and decimals, which may lead to students working with complex fractions, a fraction in the form $\frac{\frac{a}{b}}{\frac{c}{d}}$. It is important for students to interpret a complex faction as the division of two fractions.

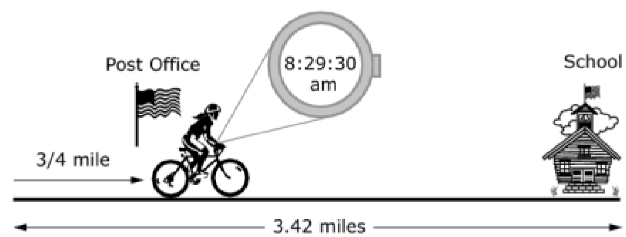
Checking for Understanding

Julia walks $\frac{1}{2}$ mile in each $\frac{1}{2}$ hour. She continues to walk at the same pace.

a) What unit rate would be needed to find how many miles Julia walked if we know the number of hours?
b) What unit rate would be needed to find how many hours Julia walked if we know how far she walked?
c) If Julia walked for $1\frac{1}{3}$ hours, how far did Julia walk?
d) If Julia walked for 5.2 miles, how long did Julia's walk take?

If a $\frac{1}{2}$ gallon of paint covers $\frac{1}{6}$ of a wall, continuing at this rate how much paint is needed for the entire wall?

Emily leaves her house at exactly 8:25 am to bike to her school, which is 3.42 miles away. When she passes the post office, which is ¾ miles away from her home, she



looks at her watch and sees that it is 30 seconds past 8:29 am.
If Emily's school starts at 8:50 am, can Emily make it to school on time without increasing her rate of speed? Show and/or explain the work necessary to support your answer.
Taken from : SBAC Mathematics Practice Test Scoring Guide Grade 7 p. 36

Analyze proportional relationships and use them to solve real-world and mathematical problems.

NC.7.RP.2 Recognize and represent proportional relationships between quantities.

a. Understand that a proportion is a relationship of equality between ratios.
   - Represent proportional relationships using tables and graphs.
   - Recognize whether ratios are in a proportional relationship using tables and graphs.
   - Compare two different proportional relationships using tables, graphs, equations, and verbal descriptions.

b. Identify the unit rate (constant of proportionality) within two quantities in a proportional relationship using tables, graphs, equations, and verbal descriptions.

c. Create equations and graphs to represent proportional relationships.

d. Use a graphical representation of a proportional relationship in context to:
   - Explain the meaning of any point (*x, y*).
   - Explain the meaning of (0, 0) and why it is included.
   - Understand that the *y*-coordinate of the ordered pair (1, *r*) corresponds to the unit rate and explain its meaning.

Clarification

In 6th grade, students worked to understand equivalent ratios and use them to solve problems. In working with ratios, students focused on using rates and scale factors to find equivalent ratios. 7th grade builds on these concepts, with the unit rate being used to determine proportionality, compare different proportional relationships, and to create different representations of the proportional relationships.

Understand that a proportion is a relationship of equality between ratios.

Student represent given proportional relationships with tables and graphs. Students determine the characteristics that remain consistent in proportional relationships, such as the unit rate and inclusion of the origin. Students determine a proportional relationship by:

- Creating tables to analyze the multiplicative relationships between the quantities (the rate) and determine their consistency.
- Creating graphs to visually verify a constant rate as a straight line through the corresponding coordinates and the origin.

As students build on the concept of proportionality, they compare different proportional relationships in various representations that may include, tables, graphs, equations, and verbal descriptions. Students compare the unit rates of the different proportional relationships. Students discuss when it is and when it is not appropriate to compare proportional relationships. For example, it is not usually appropriate to compare proportional relationships from different contexts or some situations with different units.

Students will accomplish this by examining the characteristics of each proportional relationship and describing the similarities and differences. Students may change the representation of the proportional relationships to assist with their analysis. Students use the unit rates of each proportion to

Checking for Understanding

Determine which of the following tables represent a proportional relationship? Explain your reasoning.

A.

x	0	1	3	4
y	0	5	15	20

B.

x	0	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$
y	0	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$

C.

x	0	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{3}{5}$
y	0	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$

D.

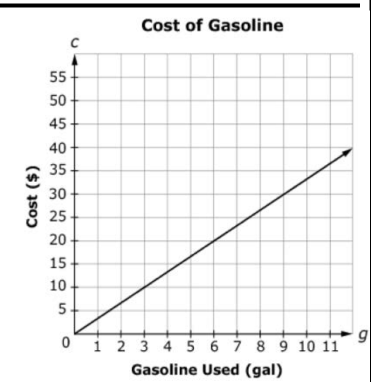
x	0	1	3	5
y	0	1	9	25

Find the unit rate, when $x = 1$, of each proportional relationship identified above and describe how you see the unit rate in the table.

The graph shows a proportional relationship between the number of gallons of gasoline used (*g*) and the total cost of gasoline (*c*).

Find the unit rate (*r*). Using the value of *r*, write an equation in the form of $c = rg$ that represents the relationship between the number of gallons of gasoline used (*g*) and the total cost (*c*).

Taken from: SBAC Mathematics Practice Test Scoring Guide Grade 7 p. 31



Analyze proportional relationships and use them to solve real-world and mathematical problems.

NC.7.RP.2 Recognize and represent proportional relationships between quantities.

a. Understand that a proportion is a relationship of equality between ratios.
   - Represent proportional relationships using tables and graphs.
   - Recognize whether ratios are in a proportional relationship using tables and graphs.
   - Compare two different proportional relationships using tables, graphs, equations, and verbal descriptions.

b. Identify the unit rate (constant of proportionality) within two quantities in a proportional relationship using tables, graphs, equations, and verbal descriptions.

c. Create equations and graphs to represent proportional relationships.

d. Use a graphical representation of a proportional relationship in context to:
   - Explain the meaning of any point (x, y).
   - Explain the meaning of (0, 0) and why it is included.
   - Understand that the y-coordinate of the ordered pair (1, r) corresponds to the unit rate and explain its meaning.

Clarification

make appropriate comparison statements and to draw conclusions. In tables and graphs, students can use common values to make comparisons.

Identify the unit rate (constant of proportionality) within two quantities in a proportional relationship using tables, graphs, equations, and verbal descriptions.

Students will expand upon their understanding of rate from 6th grade to understand that in proportional relationships every ratio in that relationship will have the same rate, or unit rate.

This unit rate is sometimes referred to as the constant of proportionality. This is because in a proportional relationship, the rate is unchanging, or constant even as the quantities increase or decrease by the scale factor. This is the most important characteristic to be identified in a proportional relationship.

Create equations and graphs to represent proportional relationships.

As each ratio produces two rates, each proportional relationship can be represented with two equations and two graphs, with the exception of ratios in a 1:1 relationship. Students will need to use the context to determine which rate, or constant of proportionality, is appropriate to each situation. Using the rate, or constant of proportionality, students can generalize the proportional relationship between the quantities to create a two-variable equation that can represent the entire proportional relationship in context. Students graph proportional relationships on the coordinate plane using the unit rate. Students will determine the appropriateness between plotting points and drawing a line based on the characteristics of the quantities involved. Students solve problems using generated equations. In 7th grade, the term slope and the slope formula are inappropriate, as the focus should remain on the multiplicative relationships.

Checking for Understanding

A landscaper is hired to take care of the lawn and shrubs around the house. The landscaper claims that the relationship between the number of hours worked and the total work fee is proportional. The fee for 4 hours of work is $140.

a) Which of the following combinations of values for the landscaper's work hours and total work fee support the claim that the relationship between the two values is proportional?

b) Write an equation that describes the proportional relationship.

A. 3 hours for $105	B. 3.5 hours for $120	C. 4.75 hours for $166.25
D. 5.5 hours for $190	E. 6.25 hours for $210.25	F. 7.5 hours for $262.50

c) What is the relationship between your answers for part a and the equation you wrote for part b?

d) What is the relationship between your non-answers for part a and the equation you wrote for part b?

The school bus driver follows the same route to pick students up in the morning and to drop them off in the afternoon. Because of traffic, the afternoon drive takes 1.5 times as long as the morning drive.

a) Write an equation that represents the relationship between the number of minutes m, of the morning drive, to the total number of minutes, t, that the bus driver spends picking up and dropping off students each day.

b) Using the unit rate, graph the equation on a coordinate plane. On your graph, should the points be connected to make a line? Explain.

Analyze proportional relationships and use them to solve real-world and mathematical problems.

NC.7.RP.2 Recognize and represent proportional relationships between quantities.

a. Understand that a proportion is a relationship of equality between ratios.
   - Represent proportional relationships using tables and graphs.
   - Recognize whether ratios are in a proportional relationship using tables and graphs.
   - Compare two different proportional relationships using tables, graphs, equations, and verbal descriptions.

b. Identify the unit rate (constant of proportionality) within two quantities in a proportional relationship using tables, graphs, equations, and verbal descriptions.

c. Create equations and graphs to represent proportional relationships.

d. Use a graphical representation of a proportional relationship in context to:
   - Explain the meaning of any point (*x, y)*.
   - Explain the meaning of (0, 0) and why it is included.
   - Understand that the *y*-coordinate of the ordered pair (1, *r*) corresponds to the unit rate and explain its meaning.

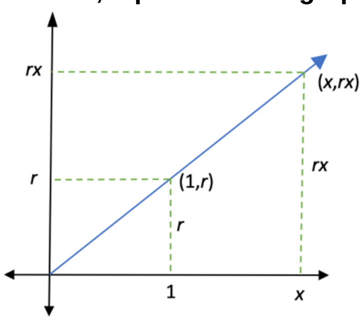
Clarification

Use a graphical representation of a proportional relationship

Students interpret the meaning of coordinates, including the origin, plotted as part of a proportional relationships. Students explain why the origin is included in all proportional relationship.

Students use the context of the situation to determine if the quantities are discrete or continuous and will only draw a line connecting the coordinates if both quantities are continuous. A continuous quantity has the ability to be continuously divided into smaller parts. For example, the number of dogs is not a continuous quantity as you cannot have ½ a dog, so a line would not be appropriate. However, a minute is a continuous quantity.

Tables, equations and graphs of proportional relationships



Students also explain how the coordinate (1,*r*) relates to the proportional relationship and its corresponding equation and table.

x (input)	*y (output)*
1	r
2	$r \cdot 2$
3	$r \cdot 3$
...	...
x	$r \cdot x$

Students recognize that the r is the multiplicative relationship between the *x* and *y* coordinates of the ordered pairs.

In the graph and in the table, 1 and *r* form a ratio. If the scale factor of *x* is multiplied to this ratio, the ratio of *x* and *rx* is produced. This means that if *x* is the input, then *rx* is the output.

This produces the proportional relationship equation, $y = rx$.

Checking for Understanding

Kell works at an after-school program at an elementary school. The table below shows how much money he earned every day last week.

Time worked	1.5 hours	2.5 hours	4 hours
Money earned	$12.60	$21.00	$33.60

Mariko has a job mowing lawns that pays $7 per hour.

a) Who would make more money for working 10 hours? Explain or show work.

b) Draw a graph that represents *y*, the amount of money Kell would make for working *x* hours, assuming he made the same hourly rate he was making last week.

c) Using the same coordinate plane, draw a graph that represents *y*, the amount of money Mariko would make for working *x* hours.

d) How can you see who makes more per hour just by looking at the graphs? Explain.

Taken from Illustrative Mathematics: Who Has the Best Job?

Select the phrase from the box to make true statements. Be prepared to justify your answer.

- In a proportional relationship, if the unit rate is _______ 1, the value of the output will be _______ the value of the input.
- When comparing proportional relationships, if the unit rate of first relationship is _______ the unit rate of the second, the value of the output of the first relationship will be _______ the value of the output of the second relationship for the same input value.

greater than equal to less than

Return to: Standards

Analyze proportional relationships and use them to solve real-world and mathematical problems.

NC.7.RP.3 Use scale factors and unit rates in proportional relationships to solve ratio and percent problems.

Clarification

In this standard, students are expected to use proportional reasoning to solve problems. Fraction and decimals may be used at all stages of the problems, and the problems may require multiple steps to find an answer. Through reasoning and repeated exposure, students may develop an algorithmic approach to solving certain problem types. These approaches and formulas are not an expectation of the standard.

This standard encompasses many problem types given the proper context. These problem types can include but are not limited to:

- converting rates to different units
- percent increase and decrease
- interpreting circle graphs

Converting Rates to Different Units

In 6th grade, students converted a single unit of measurement to a different unit of measurement. In 7th grade, students will be asked to convert both units in a rate to different units. Students are expected to use scale factors and unit rates to make the conversions. Uncommon conversion ratios should be provided. Dimensional analysis is not an expectation of this standard.

Percent Increase and Decrease

Students build upon the understanding of a percent as a ratio to solve more complex percent problems. This requires students to understand the effects on a product when multiplying a number by 1, a number less then 1, and a number greater than 1. Students are expected to use scale factors and unit rates to solve percent problems. While students should avoid “rules” or formulaic approaches, students should see the pattern and know that at percent increase or decrease is the proportional relationship between the initial value and the new value.
Students know what terms may suggest a percent increase or decrease. Some of these terms include: tax, tip, commission, fee, discount, sale, mark up, and mark down. Students may be asked to answer questions that require multiple percent increases and decreases.

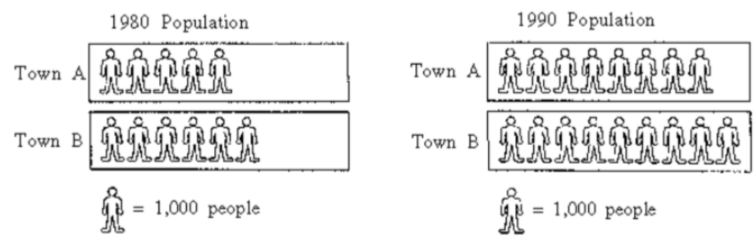
> **For example**: Abraham is taking his mother out to a restaurant for a Mother’s Day dinner. He orders a meal that cost $12.99 and his mom orders a meal for $14.79. They both order a drink for $2.75 each. For their meal, there will be a 7.5% tax and Abraham plans to leave a 15% tip for the server. How much will the entire meal cost?
> *There are multiple answers to this question depending on if the tip is determined before or after tax. The key is to listen to student reasoning.*

Checking for Understanding

Zoomy is a racing garden snail. In a snail race, the snails are given one minute to travel as far as they can. The distance traveled is then measured in feet to determine the winner. According to internet resources, a garden snail’s top speed is 0.029 mph. If Zoomy traveled at top speed, how many feet could Zoomy travel during the race? ($1\ mi = 5280\ ft$)

There were 70 employees working at a rental company. This year the number of employees increased by 10 percent. How many employees work for the rental company his year?

In 1980, the populations of Town A and Town B were 5,000 and 6,000, respectively. The 1990 populations of Town A and Town B were 8,000 and 9,000, respectively.



a) Brian claims that from 1980 to 1990 the populations of the two towns grew by the same amount. Use mathematics to explain how Brian might have justified his claim.
b) Darlene claims that from 1980 to 1990 the population of Town A grew more. Use mathematics to explain how Darlene might have justified her claim.

NAEP – Released Item (2013) **Question ID:** 1996-8M12 #5 M069601

A shirt is on sale for 40% off. The sale price is $12.

a) How much was the discount?
b) Write an equation that shows the relationship between the original price and the amount paid taking into an account an 8.5% sales tax.

Analyze proportional relationships and use them to solve real-world and mathematical problems.

NC.7.RP.3 Use scale factors and unit rates in proportional relationships to solve ratio and percent problems.

<table>
<tr><th>Clarification</th><th>Checking for Understanding</th></tr>
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Given the appropriate information, students may be asked to find the original amount, a new amount, or the percent of change.

Interpreting Circle Graphs

In 6th grade, students learned to interpret part-to-total ratios as percents. In 7th grade, students will extend this interpretation to another common part-to-total ratio, degrees. Students first used degrees to make and measure angles in 4th grade. This will be the students first exposure interpreting the measure of an angle with a ratio, in which 1 degree is 1/360 of a circle.

Students interpret a degree as being an equivalent ratio to a percent. The relationship between percents and degrees allows categorical data that form part-to-total relationships to be represented as sectors of a circle. Given appropriate information, students:

- find missing values (data, percents, or degrees)
- interpret a circle graph and use that information to solve problems

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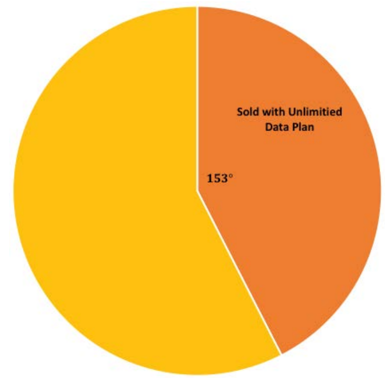
A car dealer is calculating the list price for a used car. The dealer takes the initial price of the car and adds $259 dollars for cleaning and shipping the car to the dealer. The dealer then increases that price by 25% for the dealer's profit. That price is then increased again by 10% for the salesperson's commission.

a) If a used car is initially priced $10,000, what will be the list price for this car?
b) Write an equation that shows the relationship between the initial price and the list price.

The circle graph shows the number of cell phones sold at a local store. The darker shaded portion shows the number of cell phones that were sold with an unlimited data plan. A total of 2,712 cell phones were sold.

a) Using the circle graph, approximately how many cell phones were sold with an unlimited data plan?
b) What percent of the cell phone cells are sold without an unlimited data plan?



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Return to: Standards

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers. **NC.7.NS.1** Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers, using the properties of operations, and describing real-world contexts using sums and differences.	
Clarification	**Checking for Understanding**
In 6th grade, students learned to add and subtract integers between -20 and 20 using number lines and other models. In 7th grade, students expand upon this understanding to include all rational numbers. Students understand that the properties of operations learned with whole numbers in elementary apply to rational numbers. Those properties include the identity, commutative and associative properties. Students rewrite subtraction as addition to apply properties as needed. Students apply their knowledge of addition and subtraction of rational numbers to describe real-world contexts and develop a process, their own rules, to add and subtract rational numbers.	Evaluate the following expressions: a) $5\frac{1}{2}+(-3.25)$ b) $-54.17-3.89$ c) $-283+(-35)$ d) $-4\frac{1}{4}-\left(-6\frac{1}{3}\right)$ Justin is trying to determine if he has enough money to buy a new video game. The game cost $54.79. He started the day with $210 in his bank account. Looking at his receipts, he has spent $87.35 at a clothing store, $42.79 at a party store, and $25.68 at a gas station. Does he have enough money to buy the video game? Beyond estimating, explain your answer mathematically.

Return to: Standards

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers. **NC.7.NS.2** Apply and extend previous understandings of multiplication and division. a. Understand that a rational number is any number that can be written as a quotient of integers with a non-zero divisor. b. Apply properties of operations as strategies, including the standard algorithms, to multiply and divide rational numbers and describe the product and quotient in real-world contexts. c. Use division and previous understandings of fractions and decimals. o Convert a fraction to a decimal using long division. o Understand that the decimal form of a rational number terminates in 0s or eventually repeats.	
Clarification	**Checking for Understanding**
In this standard, student multiply and divide with rational numbers. Students build upon their knowledge of multiplying and dividing with whole numbers and adding and subtracting with integers to find the product and quotient of rational numbers. In 6th grade, students focused on using a common denominator to divide fractions. In 7th grade, students will expand upon this to conceptually understand the common algorithm of multiplying by the reciprocal. **Understand that a rational number is any number that can be written as a quotient of integers with a non-zero divisor.** This standard formalizes the definition of a rational number as a quotient of integers with a non-zero divisor. Students will use this knowledge to define rational numbers. **Apply properties of operations as strategies, including the standard**	Evaluate the following expressions: a) $5(-3)$ b) $-2(10)(-3)$ c) $\frac{5}{6}(-66)$ d) $-6.3\left(-\frac{10}{21}\right)(-7)$ Two students are debating in your group. One student says that any number that can be written as a fraction is a rational number. The other student disagrees. Who is correct? If you disagree, provide a counterexample. Which of the following fractions are equivalent to $-\frac{5}{4}$? Explain. $\frac{-5}{4}, \frac{5}{-4}, -1\frac{1}{4}$

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

NC.7.NS.2 Apply and extend previous understandings of multiplication and division.

a. Understand that a rational number is any number that can be written as a quotient of integers with a non-zero divisor.
b. Apply properties of operations as strategies, including the standard algorithms, to multiply and divide rational numbers and describe the product and quotient in real-world contexts.
c. Use division and previous understandings of fractions and decimals.
   - Convert a fraction to a decimal using long division.
   - Understand that the decimal form of a rational number terminates in 0s or eventually repeats.

Clarification

algorithms, to multiply and divide rational numbers and describe the product and quotient in real-world contexts.
Students understand that the properties of operations learned with whole numbers in elementary apply to rational numbers. Those properties include the identity, commutative, associative and distributive properties and the multiplicative property of zero.
Students rewrite multiplication as division and division as multiplication and apply properties as needed. Students use the properties of operations, mathematical reasoning, and modeling to discover the rule for multiplying and dividing signed numbers. They should know facts such as:

- $-a \cdot -b = a \cdot b$
- $-\frac{p}{q} = \frac{-p}{q} = \frac{p}{-q}$

Students apply their knowledge of multiplication and division of rational numbers to describe real-world contexts and write products and quotients in an appropriate form.

Using division and previous understandings of fractions and decimals.
Students rewrite a fraction as a division problem or division problem as a fraction. Students use this knowledge to convert a fraction to a decimal, recognizing that the decimal will terminate or repeat. Students understand that when a decimal terminates or repeats, it is a rational number.

Checking for Understanding

Evaluate the following expressions:

a) $-\frac{3}{4} \div \frac{1}{2}$
b) $15 \div (-3)$
c) $-5.25 \div (-5)$
d) $\frac{2\frac{2}{3}}{-\frac{4}{3}}$

Five partners are investing in a business. The investment will cost \$21,438. One of the partners wrote this expression on a note pad, $\frac{-21{,}438}{5}$. What is the quotient and what would it represent in this situation?

A water well drilling rig has dug to a height of –60 meters after one full day of continuous use.

a) Assuming the rig drilled at a constant rate, what was the height of the drill after 15 hours?
b) If the rig has been running constantly and is currently at a height of –143.6 meters, for how long has the rig been running?

Taken from: Illustrative Mathematics "Drill Rig"

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

NC.7.NS.3 Solve real-world and mathematical problems involving numerical expressions with rational numbers using the four operations.

Clarification

Students solve multi-step problems using numerical expressions that involve addition, subtraction, multiplication, or division of rational numbers. This includes problems that involve complex fractions.
It is important for students to know common expressions that have understood grouping symbols, such as the numerator or denominator of a fraction. For example, in $\frac{4+5}{6}$, the 4+5 has an understood grouping symbol so that when being evaluated, addition would be done before the division in this expression.

Checking for Understanding

The three seventh grade classes at Sunview Middle School collected the most box tops for a school fundraiser, and so they won a \$600 prize to share among them. Mr. Aceves' class collected 3,760 box tops, Mrs. Baca's class collected 2,301, and Mr. Canyon's class collected 1,855. How should they divide the money so that each class gets the same fraction of the prize money as the fraction of the box tops that they collected?

Taken from: Illustrative Mathematics "Sharing Prize Money"

Return to: Standards

Use properties of operations to generate equivalent expressions.
NC.7.EE.1 Apply properties of operations as strategies to:
- Add, subtract, and expand linear expressions with rational coefficients.
- Factor linear expression with an integer GCF.

Clarification

In 6th grade, students added, subtracted, and expanded expressions with positive rational coefficients and factored expressions with a positive integer GCF.
Students are expected to rewrite expressions into equivalent forms by combining like terms, using the distributive property, and factoring. Students can show the created expression is equivalent to the original expression.
In 7th grade, this is limited to:
- adding and subtracting linear terms
- distribution with the product of a rational number and a linear expression
- factoring a linear expression with an integer as the greatest common factor

Checking for Understanding

Select all expressions that are equivalent to $-3.75 + 2(-4x + 6.1) - 3.25x$.
A. $7x - 2x + 8.1$ B. $8.45 - 8x - 3.25x$
C. $-1.75 - 7.25x + 6.1$ D. $-11.25x + 12.2 - 3.75$
Taken from: SBAC Mathematics Practice Test Scoring Guide Grade 6 p.18

Find the value for k that will make the following two expressions equivalent.
$-10.5x + k$ and $-2.1(5x - 3.7) + 4.1$

Select all expressions that are equivalent to $12 - 4x$.
A. $4(3 - x)$ B. $-4(x - 3)$
C. $-4(-3 + x)$ D. $2(6 - 2x)$

Use properties of operations to generate equivalent expressions.
NC.7.EE.2 Understand that equivalent expressions can reveal real-world and mathematical relationships. Interpret the meaning of the parts of each expression in context.

Clarification

Students understand that rewriting an expression into an equivalent form can provide additional information and insight into real-world and mathematical problems.
In 6th grade, students used mathematical language to identify parts of an expression. In 7th grade, students are expected to interpret the parts of an expression, such as the coefficient, constant, term, and variable, based on the context of problem.

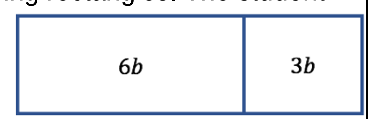
For example: Write the following as an expression: Last week, the total profit increased by 5%.
Solution: $p + 0.05p$

Rewrite the expression into an equivalent form. What do you notice?
Solution: $1.05p$*, both represent the total profit, multiplying by 1.05 produces the same value as increasing by 5%.*

It is important for students to see that rewriting an expression into the smallest length or "simplest" form is not always advantageous.

Checking for Understanding

A student was asked to find the area of the following rectangles. The student recorded the areas in the picture.



a) The student claimed the area of the entire figure was $6b + 3b$. Is the student correct?
b) Rewrite $6b + 3b$ into equivalent forms that have meaning in this picture and explain.

Return to: Standards

Solve real-world and mathematical problems using numerical and algebraic expressions, equations, and inequalities.

NC.7.EE.3 Solve multi-step real-world and mathematical problems posed with rational numbers in algebraic expressions.

- Apply properties of operations to calculate with positive and negative numbers in any form.
- Convert between different forms of a number and equivalent forms of the expression as appropriate.

Clarification

Students solve real-world and mathematical problems using a sequence of algebraic expressions. In these problems, students must express each step in the sequence using appropriate and corresponding variables. The student can then find the answer by evaluating each step in the sequence.

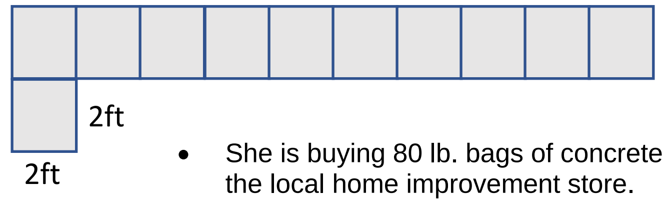
> **For example**: The cost of printing a logo on a t-shirt is based on the size of the logo and the number of colors used. The TS company charges \$.65 for each square inch on the logo, \$1.25 for each color, and \$10 for the t-shirt. Your logo is 3 inches by 1 inch and has 4 colors. You want to increase the sides of your logo by 50% for the t-shirt. How much will each shirt cost?
> *Solution: Sample expressions from the problem.*
> $.65 \cdot a + 1.25 \cdot c + 10$, *where* a *is the area of the logo in square inches and* c *represents the number of colors used*
> $1.5l \cdot 1.5w$, *where* l *is the length of the logo and* w *is the width of the logo.*
> *Need to find the area of the logo first:* $1.5 \cdot 3\ in. \cdot 1.5 \cdot 1\ in. = 6.75\ in^2$
> *Use the area of the logo in the other expression:* $.65 \cdot 6.75 + 1.25 \cdot 4 + 10 = 19.3875$. *Each t-shirt will cost \$19.39.*

The example above shows a problem that has a sequence of steps in which the answer to certain steps must be found first and substituted into the next expression to answer the question. While these problems can be answered using a purely arithmetic approach, to meet the expectation of this standard, students should write each step as an algebraic expression.
Students may choose to write algebraic expressions as multi-variable equations. The use of variables to write algebraic expressions is what distinguished is standard from NC.7.NS.3. and supports the geometry standards NC.7.G.5 and NC.7.G.6.

Students should be able to work with all rational numbers and expressions, converting to different forms, as needed, to find the answer.

Checking for Understanding

Trina is creating a small concrete sidewalk to from her driveway to her front door as seen below. Trina needs to figure out how much to budget for the concrete.



- She is buying 80 lb. bags of concrete mix that cost \$4.50 at the local home improvement store.
- Each 80 lb. bag will produce 1156 cubic inches of concrete.
- Each block measures 2 ft. by 2 ft. She wants the sidewalk to be 4 in. deep.

How much should Trina budget for concrete?
Use algebraic expressions to describe your steps to find the answer.

Katie and Margarita have \$20.00 each to spend at Students' Choice book store, where all students receive a 20% discount. Katie wants to purchase a book which normally sells for \$22.50 and Margarita wants to purchase a book which normally sells for \$22.75. With a sales tax of 10%, can Katie and Margarita buy their books?
Use algebraic expressions to describe your steps to find the answer.
Adapted from Illustrative Mathematics: Discounted Books

Solve real-world and mathematical problems using numerical and algebraic expressions, equations, and inequalities.
NC.7.EE.4 Use variables to represent quantities to solve real-world or mathematical problems.

a. Construct equations to solve problems by reasoning about the quantities.
   - Fluently solve multistep equations with the variable on one side, including those generated by word problems.
   - Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.
   - Interpret the solution in context.

b. Construct inequalities to solve problems by reasoning about the quantities.
   - Fluently solve multi-step inequalities with the variable on one side, including those generated by word problems.
   - Compare an algebraic solution process for equations and an algebraic solution process for inequalities.
   - Graph the solution set of the inequality and interpret in context.

Clarification

Students write and solve multistep one-variable equations and inequalities. In 7th grade, the variable will only be on one side of the equation or inequality. In 6th grade, students learned about the connection between an arithmetic approach, using only operations, to solve problems and an algebraic approach, using equations (see NC.6.EE.7). In 7th grade, students move from an arithmetic approach to develop an algebraic approach to solve equations and inequalities. Students describe the relationship between both approaches, paying particular attention to the sequence of steps in both approaches.

For example: You and your four friends are going to a concert. The total price for the tickets was $118.75 which includes a $10 service fee. How much did each ticket cost?

Solution: Arithmetic Approach vs Algebraic Approach

	Arithmetic		**Algebraic**
1)	Start with total, $118.75	1)	$118.75 = 5t + 10$
2)	$118.75 - 10 = 108.75$ Subtract 10 from the total	2)	$118.75 - 10 = 5t + 10 - 10$ $108.75 = 5t$
3)	$108.75 \div 5 = 21.75$ Now divide by 5 to get $21.75	3)	$108.75 \div 5 = 5t \div 5$ $21.75 = t$

The arithmetic approach is often the approach students use when they solve problems "in their head." It is important for students to see that the arithmetic approach has the same steps as the algebraic approach. (See steps 2 and 3 above.) While there is nothing wrong with the arithmetic approach, as the problems become more complex, it becomes difficult to keep track of the details. The algebraic approach allows for a routine way of solving an equation once the equation is written. Generally, the more complex the equation, the more efficient the algebraic approach becomes.

Students are expected to create multistep one variable equations and inequalities from a verbal representation and be able to describe the solution in the context of the problem. Students describe their reasoning for each step in the solving process. Students compare the solving process for equations to the solving process for inequalities. In addition, students use mathematical reasoning to explain the consequences of multiplying or dividing by negative numbers when solving inequalities.

Checking for Understanding

The youth group is going on a two-day trip to the state fair that includes a concert after the 2nd day. The trip costs $52 for each person. Included in that price is $11 for a concert ticket and the cost a pass for each day.

a) Write an equation representing the cost of the trip.
b) How much did a pass for one day cost?

Solve the following:

a) $\frac{2}{3}c - 4 = -16$ b) $\frac{t+3}{-2} = -5$
c) $10 = 2 - .25(4z - 3)$ d) $2(6 - 2a) - 4(6 - 2a) = 28$

Amy had $26 dollars to spend on school supplies. After buying 10 pens at the same price, she had $14.30 left. Write and solve an equation to determine how much each pen cost.

Florencia cannot spend more than $60 on clothes. She wants to buy jeans for $22 dollars and spend the rest on shirts. Each shirt costs $8.

a) Write an inequality to describe this situation.
b) How many shirts can she buy?

Explain why $d < -5$ and $-d > 5$ have the same solutions.

Solve the following and graph your solution on the number line:

a) $7 - x > 5.4$ b) $\frac{t-1}{-2} \leq -\frac{5}{4}$
c) $-1 > -0.5f - 5$ d) $\frac{2}{3} \leq 5 - \frac{1}{2}(4 - 3w)$

Marcus has a pool that can hold a maximum of 4500 gallons of water. The pool already contains 1500 gallons of water. Marcus begins to add more water at a rate of 30 gallons per minute.
Write an inequality that shows the number of minutes, *m*, Marcus can continue to add water to the pool without exceeding the maximum number of gallons.
Taken from: *SBAC Mathematics Practice Test Scoring Guide Grade 7 p.34*

Return to: Standards

Draw, construct, and describe geometrical figures and describe the relationships between them.

NC.7.G.1 Solve problems involving scale drawings of geometric figures by:

- Building an understanding that angle measures remain the same and side lengths are proportional.
- Using a scale factor to compute actual lengths and areas from a scale drawing.
- Creating a scale drawing.

Clarification

This standard connects geometric measurement to proportional reasoning. This standard will eventually connect to similarity in 8th grade.

Students notice that the scale factor impacts the length of line segments and the area between the scale drawing and the original drawing, while noting that the scale factor does not change the angle measurements. They also recognize how the scale factor changes in relation to the type of measurement.

Students can identify the scale factor, reproduce drawings at a different scale from a given scale and flexibly move between the actual dimensions and scaled dimensions of a drawing.

For example, students can determine the scaled dimensions of each room.

A ratio table can be used to convert from feet to inches to find the scaled dimensions of each room in the drawing starting with 20 feet = 2 inches.

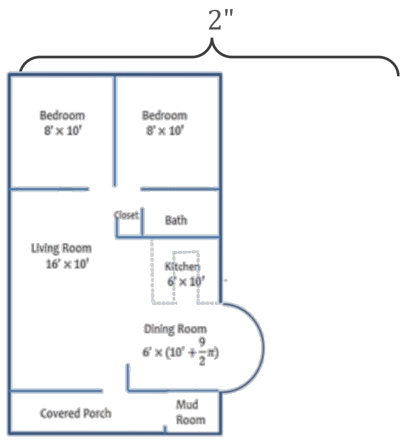
Feet	1'	2'	5'	10'	20'
Inches	0.1"	0.2"	0.5"	1"	2"

In this case, the ***living room*** has a scaled dimension of 1.6" × 1".

Additionally, a ratio table can be used to determine the scale factor. Beginning with the fact that 1' = 12" we can complete the table.

Feet	1'	2'	4'	5'	10'
Inches	12"	24"	48"	60"	120"

We can then use both tables to find the ***scale factor*** →1:120.

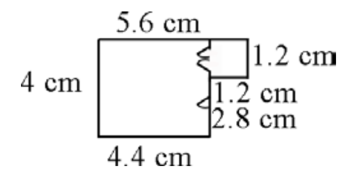


Checking for Understanding

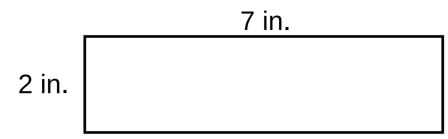
Julie shows the scale drawing of her room below.

a. If each 2 cm on the scale drawing equals 5 ft, what are the actual dimensions of Julie's room?

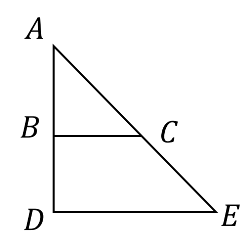
b. Reproduce the drawing at 3 times its current size.



If the rectangle below is enlarged using a scale factor of 1.5, what will be the perimeter and area of the new rectangle?



Triangle ADE is proportional to Triangle ABC. The length of $\overline{DE}$ is 20 ft.; the length of $\overline{AB}$ is 6ft. and the length of $\overline{BC}$ is 8 ft. What is the length of $\overline{AD}$?



Return to: Standards

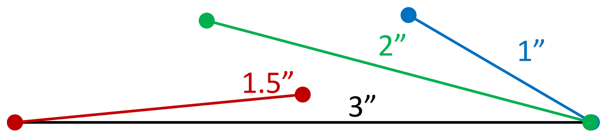
Draw, construct, and describe geometrical figures and describe the relationships between them.
NC.7.G.2 Understand the characteristics of angles and side lengths that create a unique triangle, more than one triangle or no triangle. Build triangles from three measures of angles and/or sides.

Clarification

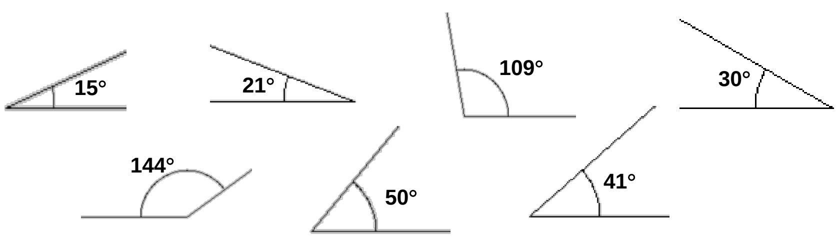
This standard focuses on the conditions that must be present for a triangle to be formed. Students should examine conditions for side lengths only, angle measurements only, and cases that include compositions of side lengths and angle measurements.

Students use a variety of tools to explore multiple cases where triangles can or cannot be formed.

Side Lengths. When determining side length characteristics for triangles, students begin to closely examine the two situations (no triangle or a unique triangle) that may arise in the formation of a triangle from 3 distinct line segments and the characteristics that determine when a triangle does or does not exist.



Angle measures. When determining angle characteristics for triangles, students use a variety of tools to explore the cases where triangles may be formed noting cases where a triangle cannot be formed, and multiple triangles can be formed. This is where students “*discover*” that the angle measures have to sum to 180° to form a triangle. ***Note:*** *Students ARE NOT expected to know the triangle sum theorem. Triangle Sum is presented in 8th grade.*



Sides lengths and angle measures. In preparation for future study of congruence, students explore situations where they are given two sides and an angle of triangle, or two angles and a side of a triangle to determine whether a unique triangle can be formed.

Checking for Understanding

Will three sides of any length create a triangle? Explain how you know which will work.

Possibilities to examine are:

a. 13 cm, 5 cm, and 6 cm
b. 3 cm, 3cm, and 3 cm
c. 2 cm, 7 cm, 6 cm

Draw a triangle with angles that are 60 degrees. Is this a unique triangle? Why or why not?

Can a triangle have more than one obtuse angle? Explain your reasoning.

Is it possible to draw a triangle with a 90° angle and one leg that is 4 inches long and one leg that is 3 inches long? If so, draw one. Is there more than one such triangle?

Draw an isosceles triangle with only one 80° angle and base angles of 50°. Is this the only possibility or can another triangle be drawn that will meet these conditions?

Return to: Standards

Solve real-world and mathematical problems involving angle measure, area, surface area, and volume.

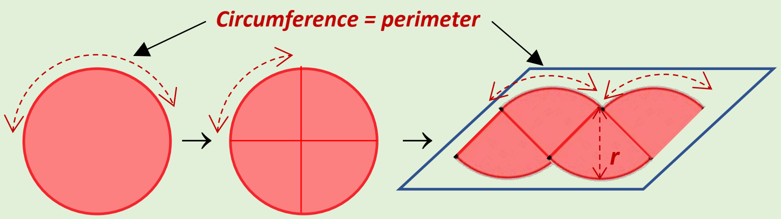
NC.7.G.4 Understand area and circumference of a circle.

- Understand the relationships between the radius, diameter, circumference, and area.
- Apply the formulas for area and circumference of a circle to solve problems.

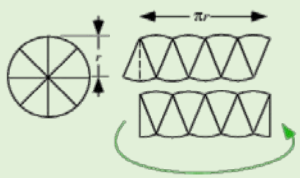
Clarification

Building on understanding of decomposing shapes intro triangles and rectangles to find area and perimeter, this standard focuses on understanding of area and circumference of circles. Beginning with the understanding that a circle is defined as a 2-dimensional figure whose boundary (circumference) consists of points equidistant from a fixed point (the center), students can decompose the figure into triangular shapes and then compose the shape into a rectangular shape.

For example, notice the circle below decomposed into triangular shapes and then composed into a parallelogram-like shape.



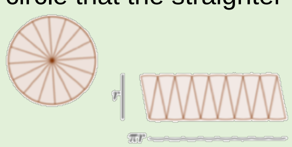
The illustration also shows the relationship between the circumference and area. As indicated above, when a circle is cut into wedges and laid out as shown, a parallelogram is the result. Half of an end wedge can then be moved to the other end a rectangular shape is the result. The height of the rectangle is the same as the radius of the circle.



http://mathworld.wolfram.com/Circle.html

Building on these understandings, students generate the formulas for circumference and area of circles.

Students also notice that the smaller the sectors in the circle that the straighter the lines appear in the constructed parallelogram.



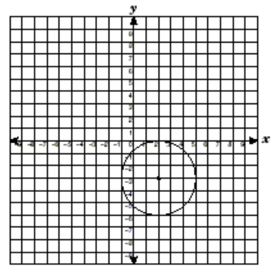
Students also use their understanding of ratios and rate to recognize that the ratio between the circumference and diameter of the circle is equivalent to the irrational number π.

Students DO NOT need to know the definition of irrational number in 7th grade.

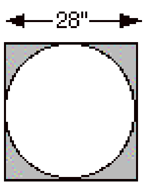
Checking for Understanding

The seventh-grade class is building a mini-golf game for the school carnival. The end of the putting green will be a circle. If the circle is 10 feet in diameter, how many square feet of grass carpet will they need to buy to cover the circle? How might someone communicate this information to the salesperson to make sure he receives a piece of carpet that is the correct size? Use 3.14 for pi.

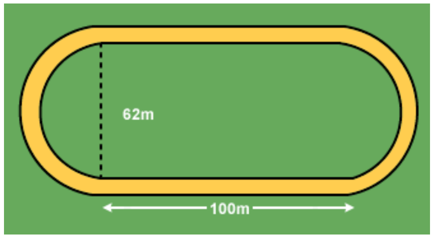
The center of the circle is at (2, -3).
What is the area of the circle?



If a circle is cut from a square piece of plywood, how much plywood would be left over?



What is the perimeter of the inside of the track.



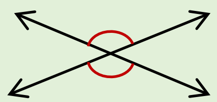
Return to: Standards

Draw, construct, and describe geometrical figures and describe the relationships between them.
NC.7.G.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve equations for an unknown angle in a figure.

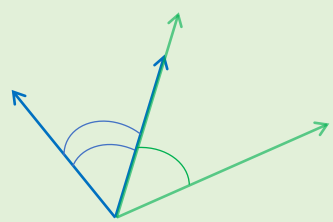
Clarification

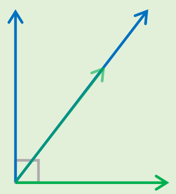
This standard focuses on angles formed by intersecting lines, vertical angles and adjacent angles, and their relationships. Students will examine patterns between these angles connecting them to vocabulary that defines their relationships.

- **Vertical angles** are opposite angles formed by intersecting lines that share a vertex. Vertical angles are <u>congruent</u> (same measure).



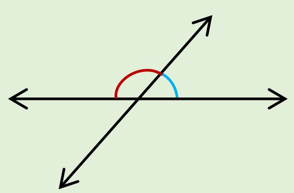
- **Adjacent angles** are two angles that have a common vertex and side.





Adjacent angles that form a right angle are <u>complementary</u> (sum to 90°).

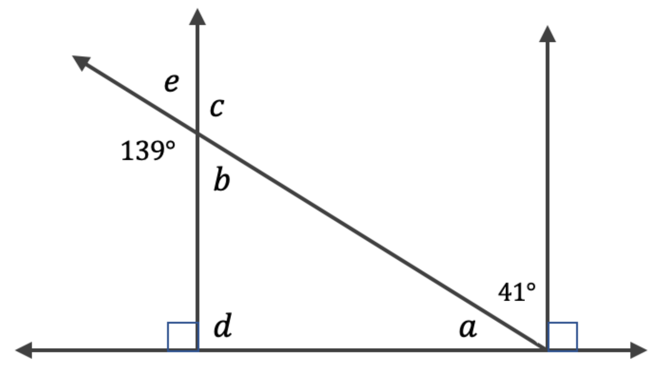
  - o **Linear pairs** are adjacent angles formed by intersecting lines. Linear pairs are <u>supplementary</u> (sum to 180°).



Students will use these relationships to build equations and solve multi-step problems (NC.7.EE.3, NC.7.EE.4).

Checking for Understanding

Use the diagram to complete the table with the measure of each labeled angle and your reasoning.



Angle (∠)	Measure (°)	Reasoning
a		
b		
c		
d		
e		

Return to: Standards

Draw, construct, and describe geometrical figures and describe the relationships between them.

NC.7.G.6 Solve real-world and mathematical problems involving:

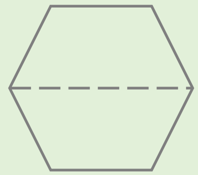
- Area and perimeter of two-dimensional objects composed of triangles, quadrilaterals, and polygons.
- Volume and surface area of pyramids, prisms, or three-dimensional objects composed of cubes, pyramids, and right prisms.

Clarification

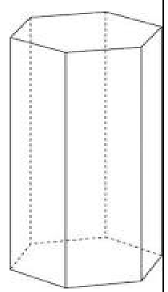
This standard focuses on extended work with composite shapes, area, perimeter, and volume from the elementary grades. Students continue to explore two- and 3-dimensional shapes.

Previously, they have calculated area and perimeter of shapes composed of rectangles and extend this learning to figures composed of triangles, quadrilaterals and polygons.

For example, students can decompose the regular hexagon to find the area or perimeter of the figure.



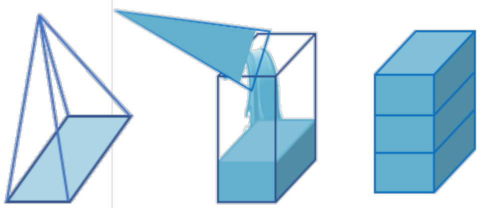
Students will further extend their work with composite shape to include volume and surface area of prisms and pyramids.



Volume of Right Prisms. Students have found the volume of right rectangular prisms with rectangular sides and bases in the elementary grades. They will extend this understanding to right prisms with polygonal bases composed of triangles, quadrilaterals and polygons. Students understand the height to be a multiple of the bases thus understanding the volume to be the product of the area of the base and the height.

Volume of Pyramids

Students recognize the volume relationship between pyramids and prisms with the same base area and height. Since it takes 3 pyramids to fill 1 prism, the volume of a pyramid is $^1/_3$ the volume of a prism.



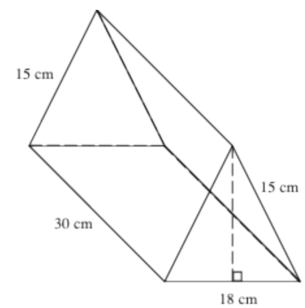
To find the volume of a pyramid, find the area of the base (*B*), multiply by the height (*h*) and then divide by three.

Therefore, $V_{pyramid} = \frac{1}{3}Bh$ *OR* $V_{pyramid} = \frac{Bh}{3}$.

Checking for Understanding

A triangle has an area of 6 square feet. The height is four feet. What is the length of the base?

Jennie purchased a box of crackers from the deli. The box is in the shape of a triangular prism.



a. If the volume of the box is 3,240 cubic centimeters, what is the height of the triangular face of the box?

b. How much packaging material was used to construct the cracker box?

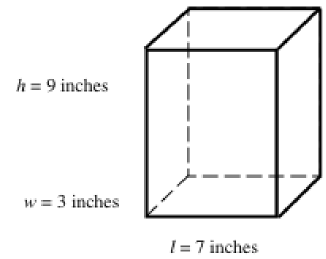
c. Explain how you got your answer.

The surface area of a cube is 96 in^2. What is the volume of the cube?

Draw, construct, and describe geometrical figures and describe the relationships between them.

NC.7.G.6 Solve real-world and mathematical problems involving:

- Area and perimeter of two-dimensional objects composed of triangles, quadrilaterals, and polygons.
- Volume and surface area of pyramids, prisms, or three-dimensional objects composed of cubes, pyramids, and right prisms.

Clarification	Checking for Understanding
Surface Area of Right Prisms. Students will build on their understanding of nets in 6th grade to develop understanding of surface area of prisms and pyramids. Students recognize that the lateral edges of a prism are rectangles and that the lateral edges of a pyramid are triangles. Students can then use what they know about area of triangles and rectangles from earlier grades to determine the area of the lateral edges and bases combined to find the surface area of the figure. Memorization of the formulas for surface area is not expected. Students should be using visualization to conceptualize surface area.	Huong covered the box to the right with sticky-backed decorating paper. The paper costs 3¢ per square inch. How much money will Huong need to spend on paper? 

Return to: Standards

Use random sampling to draw inferences about a population. **NC.7.SP.1** Understand that statistics can be used to gain information about a population by: • Recognizing that generalizations about a population from a sample are valid only if the sample is representative of that population. • Using random sampling to produce representative samples to support valid inferences	
Clarification	**Checking for Understanding**
This standard is the introduction to random sampling and how samples can be used to gather information from the population from which they are drawn, given specific conditions. Drawing representative samples from the population of interest and randomization are two such conditions. Students know the difference between a population and a sample. They should understand that a sample is a subset of the population. Therefore, inferences can only be drawn if the sample is a subset AND representative of the population. Students should know that *statistics* are the summaries that we gather from samples and *parameters* reference the population. Secondly, students understand that randomization is a condition for drawing a valid sample. Randomization reduces bias in samples. Bias in sampling interferes with the validity of inferences made based on those samples. While students are NOT expected to name the different types of bias, they should be able to articulate how an invalid sampling technique violates randomization.	**The school food service wants to increase the number of students who eat hot lunch in the cafeteria. The student council has been asked to conduct a survey of the student body to determine the students' preferences for hot lunch. They have determined three ways to select students to complete the survey. The three methods are listed below.** *Determine if each survey option would produce a random sample. If so, how do you know? If not, what condition(s) have been violated? Explain.* 1. Write all of the students' names on cards and pull them out in a draw to determine who will complete the survey. 2. Survey the first 20 students that enter the lunchroom. 3. Survey every 3rd student who gets off a bus.

Return to: Standards

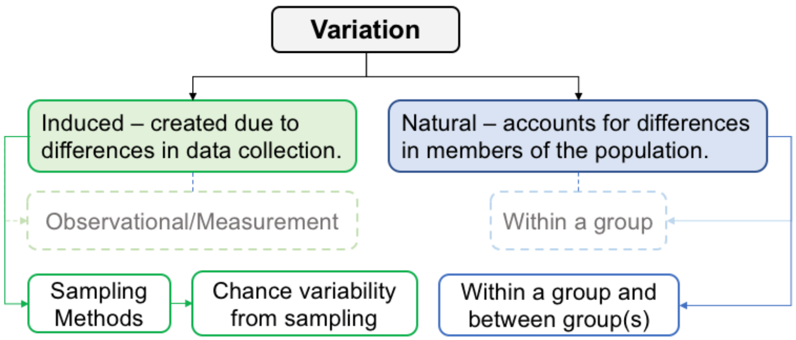
Use random sampling to draw inferences about a population.

NC.7.SP.2 Generate multiple random samples (or simulated samples) of the same size to gauge the variation in estimates or predictions, and use this data to draw inferences about a population with an unknown characteristic of interest.

Clarification

This standard requires students to collect and use *multiple* samples of data to make generalizations about a population. This can be done through actual experimentation (i.e. gathering data from samples of the population) or simulation methods (i.e. flipping a fair coin to represent 1 of 2 equally likely outcomes). Students continue to focus on statistics as a tool for explaining variability.

In 7th grade, students study induced variation from sampling methods (NC.7.SP.1) and the examination of chance processes (NC.7.SP.6, NC.7.SP.7). They continue analyzing natural variability within groups and between groups as they compare distinct populations (NC.7.SP.3, NC.7.SP.4).



Students should understand there is variation in a measure from sample to sample collected from the same population and that a sample statistic estimates a population parameter. They should also understand that a distribution of sample statistics (i.e. means, proportions, or medians) of the same size created by re-sampling can be used to estimate a population parameter by using the center and variation of the distribution to estimate an interval that the population parameter is likely within.

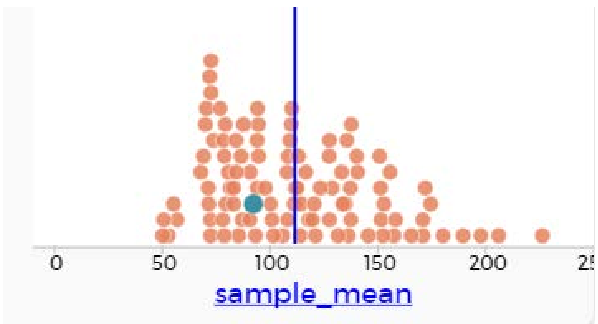
Technology is an appropriate tool to help students understand how a data distribution changes in relationship to the size of the sample or the number of samples collected. Illustrative Mathematics *Valentine Marbles* task illustrates this understanding.

Checking for Understanding

Each student in a class selected a random sample of 25 marbles from a large jar of red and white marbles and then determined the proportion of red marbles in his or her sample. The proportion in one student's sample was 0.28. The two-people sitting beside that student got sample proportions of 0.36 and 0.24. *Of the following, which gives the best explanation for the differences in the sample proportions?*

a. Sample proportions will generally differ from one random sample to another.
b. Only one of the students knew the true proportion of red marbles.
c. Two of the three students obtained bad samples.
d. Two of the three students miscalculated the percentages.

Below is a graph of a sampling distribution of 100 sample means of samples of size 25 from a sample of 199 NC high school student's responses to the question, "About how many text messages did you send yesterday?" taken from the census@school (source: http://ww2.amstat.org/censusatschool/index.cfm). The blue line represents the mean of the sampling distribution which is 111.4.



a. What does the highlighted dot in the sampling distribution represent?
b. Describe the shape, center, and spread of the sampling distribution based on its graph.
c. Consider the center and spread of the distribution of sample means to estimate what the population mean is for NC high school students number of texts sent in a day.

Return to: Standards

Make informal inferences to compare two populations.

NC.7.SP.3 Recognize the role of variability when comparing two populations.

a. Calculate the measure of variability of a data set and understand that it describes how the values of the data set vary with a single number.
   - o Understand the mean absolute deviation of a data set is a measure of variability that describes the average distance that points within a data set are from the mean of the data set.
   - o Understand that the range describes the spread of the entire data set.
   - o Understand that the interquartile range describes the spread of the middle 50% of the data.

b. Informally assess the difference between two data sets by examining the overlap and separation between the graphical representations of two data sets.

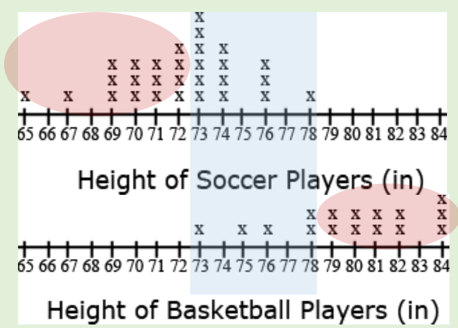
Clarification

This standard extends the understanding of comparing different data displays of one set of data (NC.6.SP.4) to making comparisons of data sets of two *distinct* populations.

Students will compute measures of variability (range, interquartile range, and mean absolute deviation) and compare the values for the two groups noting how larger values indicate more variability meaning the values are more spread out from the center of the distribution. Students understand that measures of variability are necessary to measure how far apart the centers of two different groups are to assess if they are significantly different or not.

Students will compare two data sets *visually* by examining the degree of **overlap** and **separation** in the graphs of data distributions noting similarities and differences in the context of the data.

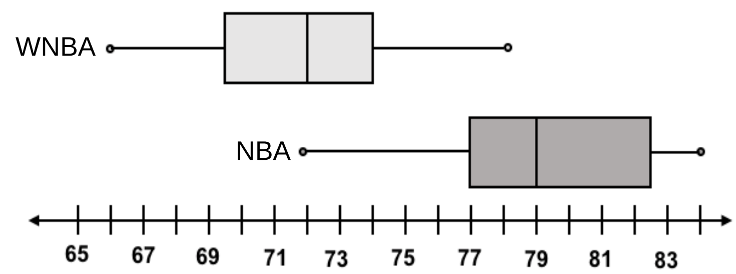
For example, in looking at the distribution of the data, observe that there is some **overlap** between the two data sets. Displaying the two graphs vertically and aligning the scales makes the comparison more *visible*.
Some players on both teams have players between 73 and 78 inches tall. However, there is a reasonable amount of **separation** between heights of soccer player and heights of basketball players. From these observations, we can infer that basketball players are *generally* taller than soccer players.



Checking for Understanding

The following data sets and boxplots represent the heights of players of a team from the WNBA and a team from the NBA, respectively.

WNBA	76, 74, 73, 78, 68, 69, 74, 74, 71, 72, 66, 70, 71
NBA	79, 80, 78, 77, 83, 83, 84, 79, 77, 82, 75, 72, 78, 73, 81, 84



a. Describe the heights of the WNBA players. How much do they vary from each other?
b. Describe the heights of the NBA players. How much do they vary from each other?
c. Box plots were used to visually compare the teams. What do the graphical displays tell us about the heights of WNBA players in comparison to the NBA players? What heights are similar? What are the differences?
d. Why is it appropriate to use box plots to compare the groups instead of dot plots or histograms?

Return to: Standards

Make informal inferences to compare two populations.

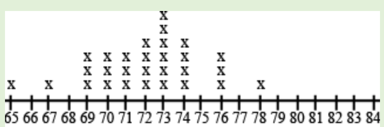
NC.7.SP.4 Use measures of center and measures of variability for numerical data from random samples to draw comparative inferences about two populations.

Clarification

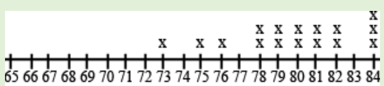
This standard focuses on the introduction of inference based on the comparisons of measures of center (NC.6.SP.5) and variability(NC.7.SP.3) of two distinct populations. Students are expected to compare two sets of data using measures of center and variability, noting which measure of center and variability are appropriate according to the shape of the distribution (i.e. mean and MAD for symmetric distributions and median and IQR for heavily skewed distributions).

Students have previously determined which measure of center to use given the shape of a single data distribution (NC.6.SP.5); this understanding is further developed in comparing data from random samples in two populations and incorporates using measures of variability to measure the differences in the measures of center of two distributions. Students should know that both distributions must be symmetrical to use the mean and mean absolute deviation (MAD) to summarize the data; otherwise, they should use the median and interquartile range.

For example, in looking at the distribution of the data, the similarities (overlap) and differences (separation) between the two data sets are easily observed. Also, the shape of each distributions is visible, which is helpful in determining the measures of center and variability to use in the analysis. Since the height of basketball players is skewed, the median and IQR should be used to compare the data sets.



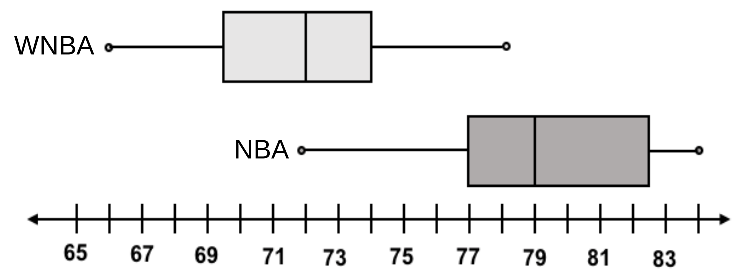
Height of Soccer Players (in)



Height of Basketball Players (in)

Checking for Understanding

Box plots are a good tool to use to visually compare range and IQR when comparing data sets. In general, no overlap in the IQR of data sets indicates that there is likely a significant difference in the centers. This meaning that the heights there is a significant difference in the heights of male and female professional basketball players. Explain how the graphical displays below confirm that NBA player heights are generally higher than WNBA heights.



Return to: Standards

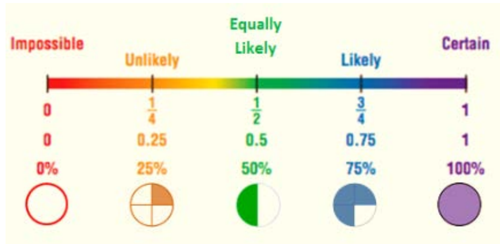
Investigate chance processes and develop, use and evaluate probability models.

NC.7.SP.5 Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring.

Clarification

This standard introduces students to ***probability*** associated with chance events. Students recognize that the probability of any single event can be expressed using terminology like impossible, unlikely, likely, or certain or as a number between 0 and 1, inclusive, with numbers closer to 1 indicating greater likelihood.

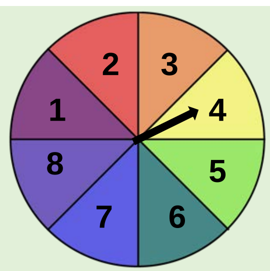
Students understand that probabilities are expressed as ratios of the number of times that an event occurs to the total number of trials that are conducted. Students know that probabilities can be represented by a fraction, decimal, or a percent.



Students should be able to describe the likelihood based on the proportion of successes for the event. Students also understand the relationship between an event and the compliment of the same event.

Students understand the likelihood of simple events and the connection to the tool being used.

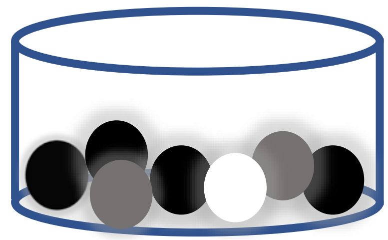
For example, the spinner has 8 sections, therefore landing on any one color has the same likelihood (unlikely→$\frac{1}{8} = .125 \rightarrow 12.5\%$), however landing on a value less than 7 is more likely ($\frac{6}{8} = \frac{3}{4} = .75 \rightarrow 75\%$) than a number smaller than 7 ***OR*** landing on a number greater than 10 is impossible ($\frac{0}{10} = 0$).



Students can use a variety of random experiments to perform simple probability experiments by hand to quantify and interpret likeliness of an event occurring.

Checking for Understanding

The container below contains 2 gray, 1 white, and 4 black marbles. *Without looking, if Eric chooses a marble from the container, will the probability be closer to 0 or to 1 that Eric will select a white marble? A gray marble? A black marble? Justify each of your predictions.*



There are three choices of jellybeans – grape, cherry and orange. *If the probability of getting a grape is* $\frac{3}{10}$ *and the probability of getting cherry is* $\frac{1}{5}$, *what is the probability of getting orange?*

Return to: Standards

Investigate chance processes and develop, use and evaluate probability models.
NC.7.SP.6 Collect data to calculate the experimental probability of a chance event, observing its long-run relative frequency. Use this experimental probability to predict the approximate relative frequency.

Clarification

The focus of this standard is on ***relative frequency***, which is the observed proportion of successful outcomes compared to the total number of trials for chance events. This standard connects probability models to chance events related to sampling and sampling variability.

Students recognize that individual experimental results may vary for each separate trial, which may also differ from the long run probability.

For example, the relative frequency table shows the result for tossing a coin 50 times. Recognizing that this is one sample, students can account for the difference from the *theoretical* probability of getting heads or tails (.50 or 50%) based on the variation attributed to the *experiment*.

	Frequency	*Relative Frequency*
Heads	27	$\frac{27}{50} = .54$
Tails	23	$\frac{23}{50} = .46$
Total	50	$\frac{50}{50} = 1$

This standard is intended to use experimentation to show that over a large number of trials that relative frequencies for ***experimental probabilities*** become closer to the ***theoretical probabilities***.

Students should make predictions before conducting the experiment, run trials of the experiment and refine their conjectures as they run additional trials. It is appropriate to use graphing calculators or computer simulation programs to collect large amounts of data on chance events.

Additionally, digital software can be used to conduct a large number of trials. The following are examples of online simulators that can be used:

- Interactive Coin Toss – Shodor
- GeoGebra Coin Flip Simulation

Checking for Understanding

A bag contains 100 marbles, some red and some purple. Suppose a student, without looking, chooses a marble out of the bag, records the color, and then places that marble back in the bag. The student has recorded 9 red marbles and 11 purple marbles. *Using these results, predict the number of red marbles in the bag.*

(Adapted from SREB publication *Getting Students Ready for Algebra I: What Middle Grades Students Need to Know and Be Able to Do)*

Design a Probability Experiment:
For example, give each pair of students a bag that containing 4 green marbles, 6 red marbles, and 10 blue marbles.

1. Each group performs 50 pulls, recording the color of marble drawn and replacing the marble into the bag before the next draw.
2. Students summarize their data as *experimental* probabilities and make conjectures about *theoretical* probabilities. ***How many green draws would be expected if 1000 pulls are conducted? 10,000 pulls?***
3. Students record their data in a relative frequency table as they compile their results with the class. ***How did the relative frequencies change?***
4. ***Optional:*** Students can create another scenario with a different ratio of marbles in the bag and make a conjecture about the outcome of 50 marble pulls with replacement.

Return to: Standards

Investigate chance processes and develop, use and evaluate probability models.
NC.7.SP.7 Develop a probability model and use it to find probabilities of simple events.
a. Develop a uniform probability model by assigning equal probability to all outcomes and use the model to determine probabilities of events.
b. Develop a probability model (which may not be uniform) by repeatedly performing a chance process and observing frequencies in the data generated.
c. Compare theoretical and experimental probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

Clarification

This standard focuses on the development and understanding of a ***probability model.*** Students understand that the ***sample space*** and related probabilities define the probability model for a random circumstance. Students also understand the difference between ***uniform probability models*** (all outcomes have the same probability) and ***non-uniform probability models*** (outcomes with different probabilities).

For example, given a cube with Event A = roll a letter (A – F), the probability model has a sample space (S) of {A, B, C, D, E, F} where $P(A_A) = P(A_B) = P(A_C) = P(A_D) = P(A_E) = P(A_F) = \frac{1}{6}$. This describes a uniform probability model.

Given the same cube with Event B = roll a colored letter, the probability model has a sample space (S) of {green letter, black letter} where $P(B_{green}) = \frac{4}{6} = \frac{2}{3}$ and $P(B_{black}) = \frac{2}{6} = \frac{1}{3}$. This represents a non-uniform probability model.

Note: *The top and bottom of the cube have black letters on a green surface (A and F) and the lateral edges have green letters on a black surface (B, C, D and E).*

Using theoretical probability, students predict frequencies of outcomes, then compare the theoretical and experimental probabilities from a model and explain possible sources of noted variation between the probabilities. Students recognize an appropriate design to conduct an experiment with simple probability events, understanding that the experimental data give realistic estimates of the probability of an event but are affected by sample size.

Students should be provided with multiple opportunities to perform probability experiments and to compare the results to theoretical probabilities. Critical components of each experimental process:

- Making predictions about the outcomes by applying the principles of theoretical probability;
- Comparing the predictions to the outcomes of the experiments;
- Replicating the experiment and continuing to compare results.

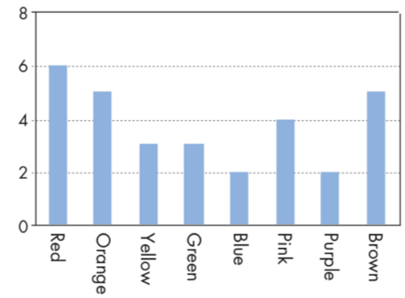
Experiments can be conducted using technology or physical objects (i.e. bag pulls, spinners, number cubes, coin tosses, colored chips, etc.)

Checking for Understanding

Robert's mother lets him pick one candy from a bag. He can't see the candies. The number of candies of each color in the bag is shown in the following graph.

What is the probability that Robert will pick a red candy? Explain.

A. 10%
B. 20%
C. 25%
D. 50%



PISA Mathematics Sample Task (2009) – Question 14.1 (Page 115)

Look at the shirt you are wearing today and determine how many buttons it has. Then complete the following table for all the members of your class.

	No Buttons	One or Two Buttons	Three or Four Buttons	More Than Four Buttons
Male				
Female				

Suppose each student writes his or her name on an index card, and one card is selected randomly.
a. What is the probability that the student whose card is selected is wearing a shirt with no buttons?
b. What is the probability that the student whose card is selected is female and is wearing a shirt with two or fewer buttons?

Illustrative Mathematics – How Many Buttons?

Return to: Standards

Investigate chance processes and develop, use and evaluate probability models.

NC.7.SP.8 Determine probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.
b. For an event described in everyday language, identify the outcomes in the sample space which compose the event, when the sample space is represented using organized lists, tables, and tree diagrams.
c. Design and use a simulation to generate frequencies for compound events.

Clarification

This standard focuses on the use of organized lists or tables and tree diagrams to determine the probability of ***compound events***. Therefore, students are expected to extend their understanding of simple events to that of compound events. They should compare and contrast simple and compound events both orally and in writing and draw on context to demonstrate their understanding.

For example, when flipping a coin two times, a student should be able to determine the sample space based on what they know about the outcomes for each flip.

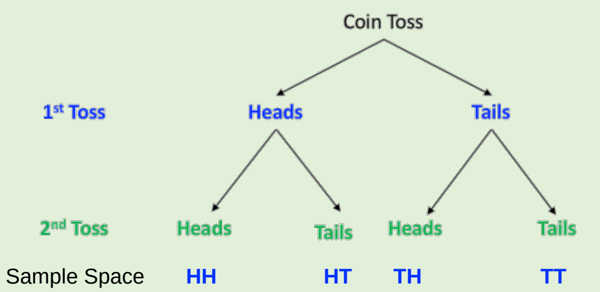
Organized List

HH (heads both flips)
HT (heads then tails)
TH (tails then heads)
TT (tails both flips)

Table

1st Toss	2nd Toss	Sample Space
H	H	HH
H	T	HT
T	T	TT
T	H	TH

Tree Diagram



Students are also expected to know and understand how to determine the sample space of compound events and explain how the sample space is used to find the probability of compound events (with or without replacement).

Additionally, students can design simulations to collect data for compound events to generate frequencies of compound events for the purpose of approximating probabilities of compound events.

Checking for Understanding

A fair coin will be tossed three times. What is the probability of getting two heads and one tail for the three tosses in any order?

A. Show all possible arrangements of the letters in the word FRED using a tree diagram.
B. If each of the letters is on a tile and drawn at random, what is the probability of drawing the letters F-R-E-D in that order?
C. What is the probability that a "word" will have an F as the first letter?

Return to: Standards